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EXISTENCE OF A SOLUTION OF FREDHOLM-VOLTERRA INTEGRAL EQUATIONS VIA FIXED POINT THEOREMS

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Abstract

In this paper we present fixed point theorems to obtain existence for integral equations. For this purpose developed fixed point theorems to nonlinear integral equations are also discussed. In particular, our results encompass the analogues of Brouwer's and Schouder's fixed point theorems for closed ball, convex subset and continuous mappings and some of their generalizations.

Keywords**:** Fixed point theorem, contraction mapping**,** integral equations.

Introduction

Over the last several decades, nonlinear functional analysis has been an active area of research in mechanics, elasticity, and fluid dynamics and so on. Functional analysis deals with classes of functions was founded by Banach, Frechet Gelfund, Housdorff, Hilbert, Mazur, Van Neumann, Riesz, Stone and a number of other dedicated workers. In 1922, Stefan Banach proved a constructive fixed point theorem which is known as Banach Contraction Principle. In this paper we will discuss about this fixed point theorem which relates contraction of functional operators.

Fixed point theory is an utmost prior tool in the field of nonlinear functional analysis to solve the nonlinear differential and integral equations. In most cases the differential and integral equations are transformed into an equivalent operator equation involving integral operators and then appropriate fixed point theorem are invoked to prove the existence of desire solution by recasting the operating equations into fixed point equations.

Fixed point theorem and it's application on differential and integral equations have been studied by several authors like (Boyd and Wong 1969; Burton and Kirk 1998;

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Dhage, 2002; Dhage and Ntouyas, 2003; Krasnoselskii, 1958), (Reinermann, 1971) and the reference given therein. After the classical results by Banach (1922) provided a subsequently new contractive mapping to prove the fixed point theorems. Since then there have been many theorems emerged as generalization under various contractive conditions. The interested reader who wants to know more about this matter is recommended to go deep into the survey articles by (Regan, 1996; Joshi and Bose 1985; Smart, 1980), and into the references therein.

In this paper, we used Brouwer's fixed point theorem which stated as, the unit ball $B \subset \mathbb{R}^n$ has fixed point property. Due to some limitations of Brouwer's theorem we emphasis on Schauder's fixed point theorem which encompasses convex subset, normed linear space and continuous mapping. Finally to investigate the existence of solutions of Fredholm-Volterra integral equations we first recall some basic definitions and elementary results. Let X denote a Banach space with a norm $\| \, . \, \|$. Let $a \in X$ and let r be a positive real number. Then by $B_r(a)$ and $\overline{B_r}(a)$ we respectively denote an open and a closed ball in X centered at the point $a \in X$ and of radius r. A mapping $T: X \to X$ is said to be Lipschitz if there exists a real number $k \geq 0$ such that for all $x, y \in X$, we have

$$\|Tx - Ty\| \leq k\|x - y\|$$

T is said to be contraction if $k < 1$ and nonexpansive if $k = 1$. T is said to be contractive if for all $x, y \in X$ and $x \neq y$ have

$$\|Tx - Ty\| \leq \|x - y\|.$$

Let (X, d) be a complete metric space and $T: X \to X$ be a contraction mapping. Then T has a unique fixed point x_0 and for each $x \in X$ we have

$$\lim_{n \to \infty} T^n\,(x) = x_0.$$

Moreover for each $x \in X$, we have

$$d(T^n(x), x_0) \leq \frac{k^m}{1-k} d(T(x), x).$$

1. Mathematical Preliminaries

Definition 1.1 (Joshi and Bose, 1985): Let (X, d) be a metric space and $T: X \to X$ be a map. An element $x \in X$ is said to be a fixed point of the mapping if $Tx = x$.

Definition 1.2 (Joshi and Bose. 1985): A metric space (X, d) is said to be complete if every Cauchy sequence in X converges to a point in X.

Definition 1.3 (Joshi and Bose, 1985): A complete normed linear space is called a Banach space.

Definition 1.4 (Joshi and Bose. 1985): A subset K of a convex metric space X is said to be convex if $W(x,y,\lambda) \in K,\ \ x,y \in K$ and $\lambda \in (0,1]$.

The local version of well known fixed point theorem is stated below.

Theorem 1.1 (Dugundji and Granas, 1982): Let $T: \overline{B_r}(a) \to X$ be the contraction with contraction constant α. If T satisfies

$$\|\alpha - T\alpha\| \leq (1-\alpha)r$$

for some $a \in X$ and $r > 0$, then T has a unique fixed point in $\overline{B_r}(a)$.

Theorem 1.2 (Boyd and Wong, 1969): Let S be a closed convex and bounded subset of a Banach Space X and let $T: S \to S$ be a nonlinear contraction. Then T has a unique fixed point x^* and the sequence $\{T^n x\}$ of successive iterations converges to x^* for each $x \in X$.

The operator $T: X \to X$ is called compact if $T(X)$ is a compact subset of X. Similarly $T: X \to X$ is called totally bounded if $T(S)$ is a totally bounded set in X, for every bounded set S of X. Finally a completely continuous operator $T: X \to X$ is one which is continuous and totally bounded (Dhage, 2003)

A fixed point theorem of Schaefer (Regan, 1996), concerning the completely continuous operator is stated below.

Theorem 1.3 (Regan, 1996): Let $T: X \to X$ be a completely continuous operator. Then either

(a) the operator equation $x = \lambda Tx$ has a solution for $\lambda = 1$, or

(b) the set $\varepsilon = \{u \in X\ |u = \lambda Tu\}$ is unbounded for $\lambda \in (0,1)$.

Later Burton and Kirk (Burton and Kirk, 1998) combined Theorem 1.2 and Theorem 1.3 and proved the following fixed point theorem:

Theorem 1.4 (Burton and Kirk, 1998): Let $S, T: X \to X$ be two operators satisfying:

(i) S is a contraction and

(ii) T is completely continuous.

Then either

(a) the operator equation $Sx + Tx = x$ has a solution, or

(b) the set $\varepsilon = \left\{u \in X \mid \lambda S\left(\frac{u}{\lambda}\right) + \lambda T u = u\right\}$ is unbounded for $\lambda \in (0,1)$.

The local version of the famous Schauder's fixed point theorem may be given as follows.

Theorem 1.5 (Dhage, 2003): Let $a \in X$ and let r be a positive real number. If $T: \overline{B_r}(a) \to \overline{B_r}(a)$ be a completely continuous operator, then T has a fixed point.

Theorems 1.1 and 1.5 have been extensively used in the literature for proving the existence of the solution of nonlinear differential and integral equations in the neighborhood of a point in the function space.

The next important topological fixed point theorem in its original form is stated like the following.

Theorem 1.6 (Krasnoselskii, 1958): Let S be a closed convex and bounded subset of X and let $A, B : S \to X$ be two operators such that

(a) A is a contraction,

(b) B is completely continuous and

(c) $Ax + By \in S$ for all $x, y \in S$.

Then the operator $Ax + Bx = x$ has a solution in S.

Theorem 1.6 is helpful in the study of nonlinear integral equations of mixed type which arise as a inversion of the perturbed differential equations and so it has attracted the attention of several authors (Burton and Kirk 1998) and the reference therein. Attempts have been made to improve or generalize Theorem 1.6 in the course of time by weakening the hypothesis (a) or (b) or (c) of it. We focus our attention on the hypothesis (c) of the Theorem 1.6. The following reformulation of Theorem 1.6 is note-worthy and is proved in (Reinermann, 1971).

Theorem 1.7 (Reinermann, 1971): Let S be a closed convex and bounded subset of a Banach space X and let $A, B : S \to X$ be two operators such that

(a) A is a contraction,

(b) B is completely continuous and

(c) $Ax + Bx \in S$ for all $x \in S$.

Then the operator $Ax + Bx = x$ has a solution.

The following two more reformulation of Theorem 1.7 have been recently obtained in the literature by (Regan, 1996) and (Burton, 1998) under some weaker hypothesis (c) thereof.

Theorem 1.8 (Regan, 1996 and Burton, 1998): Let S be a closed convex and bounded subset of X and let $A, B : X \to X$ be two operators such that

(a) $A + B: S \to X$,

(b) $A + B$ is condensing and

(c) If $\{(x_j, \lambda_j)\}$ is a sequence in $\partial S \times [0,1]$ converging to (x, λ) with $x = \lambda(A + B)x$ and $0 < \lambda < 1$, where ∂S is the boundary of S, then $\lambda_j(A + B)x \in S$ for long j.

The measure of noncompactness and condensing mapping require a high technicalities which a nonspecialist working in the field of nonlinear problems may find difficulty to tackle it with and therefore, Theorem 1.6 is generally used as a handy tool in applications to perturbed nonlinear equations. For the more details of condensing maps the readers are referred to (Zeidler, 1985).

Theorem 1.9 (Burton, 1998): Let S be a closed convex and bounded subset of X and let $A : X \to X$ and $B : S \to X$ such that

(a) A is a contraction,

(b) B is completely continuous and

(c) $\{x = Ax + By \; for \; all \; y \in S\} \implies x \in S$.

Then the operator $Ax + Bx = x$ has a solution.

2. Fixed point theorems

In this section we shall discuss the extension of the theorem that was stated before as mathematical preliminaries.

Theorem 2.1 (Dhage, 2003): Let $\in X$, r a positive real number. Let $A : X \to X$ and $B : \overline{B}_r(a) \to X$ be two operators such that

(a) A is a contraction with a contraction constant α.

(b) B is completely continuous, and

(c) $\|a - (Aa + By)\| \leq (1 - \alpha)r$ for all $y \in \overline{B}_r(a)$.

Then the operator equation $Ax + Bx = x$ has a solution in $\overline{B}_r(a)$.

Proof. The proof follows by applying Theorem 1.5 to the operator T defined by

$$T = (I - A)^{-1}B.$$

First we claim that T is well defined and

$$T: \overline{B}_r(a) \to \overline{B}_r(a). \tag{2.1}$$

Notice that $(I - A)^{-1}$exists and is continuous on X in view of hypothesis (a). Hence the mapping T in (2.1) is well defined. Let $y \in X$ and define a mapping A_y on $\overline{B}_r(a)$ by

$$A_y(x) = Ax + By.$$

We show that $A_y$is a contraction on $\overline{B}_r(a)$. For any $x_1, x_2 \in \overline{B}_r(a)$,we have

$$\|A_y(x_1) - A_y(x_2)\| = \|Ax_1 - Ax_2\| \leq \alpha\|x_1 - x_2\|,$$

where $0 < \alpha < 1$, and so $A_y$is a contraction on $\overline{B}_r(a)$. Again by hypothesis (c),

$$\|a - A_y(a)\| = \|a - (Aa + By)\| \leq (1 - \alpha)r.$$

Hence an application of Theorem 1.1 yields that there is a unique point x^*in $\overline{B}_r(a)$such that

$$A_y(x^*) = x^*, \;\; i.e., \;\; x^* = Ax^* + By \;\text{ or }\; (1 - A)x^* = By \tag{2.2}$$

Now applying $(1 - A)^{-1}$on both the sides of (2.2), we obtain

$$x^* = (1 - A)^{-1}By$$

Or equivalently, $Ty = x^*$. This guarantees the claim (2.1). The operator T, which is the composition of a continuous and a completely continuous operator, is completely continuous. Now the desired conclusion follows by an application of Theorem 1.5. This completes the proof.

Taking $a = 0$, the origin of X, in Theorem 2.1 we obtain

Corollary 2.1: Let $A: X \to X$and $B: \overline{B}_r(0) \to X$ be two operators such that

(a) A is a contraction with the contraction constant α

(b) B is a completely continuous, and

(c) $\|A0 + By\| \leq (1 - \alpha)r$ for all $y \epsilon \overline{B}_r(0)$.

Then the operator equation $Ax + Bx = x$ has a solution in $\overline{B}_r(0)$.

Next we prove a local version of the following fixed point theorem of (Nashed and Wong, 1969), which is also useful for applications in the theory of differential and integral equations.

Theorem 2.2 (Nashed and Wong, 1969): Let S be a closed convex and bounded subset of X and let $A, B: S \to X$ be two operators such that

(a) A is linear and bounded and there is a positive integer p such that A^p is a contraction,

(b) B is a completely continuous, and

(c) $Ax + By \epsilon S$ for all $x, y \epsilon S$.

Then the operator equation $Ax + Bx = x$ has a solution.

Theorem 2.3: Let $A: X \to X$and $B: \overline{B}_r(a) \to X$ be two operators such that

(a) A is linear and bounded and there exists $a, p \epsilon N$such that $A^p$is a contraction with contraction constant α,

(b) B is completely continuous, and

(c) $\|a - A^p a\| + \left(\frac{1 - \|A\|^p}{1 - \|A\|}\right) \|B_y\| \leq (1 - \alpha)\, r \; for \; all \; y \, \epsilon \, \overline{B}_r(a).$

Then operator equation $Ax + Bx = x$ has a solution in $\overline{B}_r(a)$.

Proof. The proof involves applying Theorem 1.5 to the operator T defined by

$$T = (1 - A)^{-1} B. \tag{2.3}$$

We claim that T is well defined and

$$T: \overline{B}_r(a) \to \overline{B}_r(a) \tag{2.4}$$

Now

$$(1 - A)^{-1} = 1 + A + A^2 + \cdots$$

$$= (1 - A)^{-1} \left(\sum_{j=0}^{p-1} A^j \right)$$

Clearly $(1-A)^{-1}$ exists since $A^p$is a contraction. Also, the operator $\left(\sum_{j=0}^{p-1} A^j\right)$ and consequently the operator T is well defined. Now we shall prove the claim (2.4).

Let $y \epsilon \overline{B}_r(a)$be fixed and define a mapping A_y on $\overline{B}_r(a)$ by

$$A_y(x) = Ax + By.$$

We shall show that $A_y^p$is a contraction. Let $x_1, x_2 \epsilon \overline{B}_r(a)$. Then by the hypothesis (a),

$$\|A_y(x_1) - A_y(x_2)\| = \|Ax_1 - Ax_2\|.$$

Again

$$\|A_y^2(x_2) - A_y^2(x_2)\| = \|A_y(A_y(x_1)) - A_y(A_y(x_2))\|$$
$$= \|A^2x_1 - A^2x_2\|$$

Similarly

$$\|A_y^p(x_1) - A_y^p(x_2)\| = \|A_y^p(x_1) - A_y^p(x_2)\| \leq \alpha\|x_1 - x_2\|,$$

Where $0 \leq \alpha < 1$. As a result, A_y is a contraction on $\overline{B}_r(a)$.

Now

$$A_y(x) = Ax + By$$
$$A_y^2(x) = A_y\big(A_y(x)\big)$$
$$= A_y(Ax + By) + By$$
$$= A(Ax + By) + By$$
$$= A^2x + ABy + By$$

Similarly

$$A_y^3(x) = A_y(A_y^2x)$$
$$= A(A^2x + ABy + By)$$
$$= A^3x + A^2By + ABy + By$$

By induction,

$$A_y^p(x) = A^px + A^{p-1}By + A^{p-2}By + \cdots + By.$$

Therefore,

$$\|a - A^p y(a)\| = \left\|a - A^p a - \left(\sum_{j=0}^{p-1} A^j\right) By\right\|$$

$$\leq \|a - A^p a\| + \left\|\sum_{j=0}^{p-1} A^j\right\| \|By\|$$

$$\leq \|a - Aa\| + \left(\sum_{j=0}^{p-1} \|A^j\|\right) \|By\|$$

$$\leq \|a - A^p a\| + \left(\frac{1 - \|A\|^p}{1 - \|A\|}\right) \|By\|$$

$$\leq (1 - \alpha) r.$$

An application of Theorem 1.1 yields that there is a unique point $x^* = \overline{B}_r(a)$ such that
$A_y(x^*) = x^*,\ Ax^* + By = x^*$

or, equivalently $(1 - A)^{-1} By = x^*$, i.e., $Ty = x^*$.

This proves the claim (2.3). Since A is linear bounded, it is continuous, and as a result $(1 - A)^{-1}$ is continuous on X. Now the operator T, which is a composition of a continuous and a completely continuous operator, is completely continuous on $\overline{B}_r(a)$. Hence the conclusion follows by an application of Theorem 1.9.

3. Fredholm-Volterra Integral Equations

In this section we shall discuss the existence of integral equation particularly Fredholm-Volterra equations.

$$x(t) = f\big(t, x(t)\big) + \lambda \int_a^b k_1(t,s) g\big(s, x(s)\big) ds + \int_0^t k_2(t,s) h(s, x(s)) ds \qquad (3.1)$$

Where $f, g : [a,b] \to \mathbb{R}$, $h: X \times X \to \mathbb{R}$, $k_1, k_2 : [a,b] \times [a,b] \to \mathbb{R}$ are continuous with Banach space X. The integral equation (3.1) will be studied under the following assumptions:

(i) $\left|g\big(t,x(t)\big)-g\big(t,y(t)\big)\right| \le \mu\|x(t)-y(t)\|, \forall\, x(s),y(s) \in \overline{B}_r$,
where $\overline{B}_r = \{x(t): |x(t)| \le r\}$ and $\mu \ge 0$ a constant;

(ii) $\left|h\big(t,x(t)\big)-h\big(t,y(t)\big)\right| \le \xi\|x(t)-y(t)\|, \forall\, x(t),y(t) \in \overline{B}_r$ and $\xi \ge 0$ a constant;

(iii) $|f(t,x(t))-f(t,y(t))| \le r\|x(t)-y(t)\|, \forall\, x(t),y(t) \in \overline{B}_r$ and $r \ge 0$ a constant;

(iv) $k_1(t,s)$ and $k_2(t,s)$ are such that S and T are continuous operators from $BC[0,\infty)$ into itself.

Then there exists a unique solution of (3.1) provided $(\lambda\mu k_1^* - \xi k_2^* + r) < 1$ and

$$\left|f\big(t,x(t)\big)\right| + \lambda k_1^*|g(t,0)| + k_1^*|h(t,0)| \le r(1-\lambda\mu k_1^* - \xi k_2^*).$$

Theorem 3.1: Let X be a Banach space and suppose (i)-(iv) hold. Then (3.1) has a solution in on $BC[0,\infty)$ on X.

Proof : Let us define two operators S and T from $BC[0,\infty)$ into itself by

$$Tx(t) = \lambda \int_a^b k_1(t,s) g(t,x(s))ds \tag{3.2}$$

$$Sx(t) = f\big(t,x(t)\big) + \int_0^t k_2(t,s)h(t,x(s))ds \tag{3.3}$$

It is sufficient to show that the mapping T defined by (3.2) is a contraction. For two continuous functions $x(t), y(t) \in BC[0,\infty)$, we have

$$|Tx(t)-Ty(t)| = \left|\lambda \int_a^b k_1(t,s)g(s,x(s))ds - \lambda \int_a^b k_1(t,s)g(s,y(s))ds\right|$$

$$= |\lambda| \left|\int_a^b k_1(t,s)g(s,x(s))ds - \int_a^b k_1(t,s)g(s,y(s))ds\right|.$$

$$\le |\lambda| \left|\int_a^b k_1(t,s)\right| \left|g\big(s,x(s)\big) - g\big(s,y(s)\big)\right|ds$$

$$\le |\lambda| M\mu\, \|x(t)-y(t))\| \int_a^b ds$$

where $= sup_{x\in[a,b]} \int_a^b k_1(t,s)$.

Thus $|Tx(t)-Ty(t)| \le |\lambda|M\mu \left\|x\big(t\big)-y\big(t\big)\right\|(b-a)$

By induction we can show that,

$$|T^m x(t) - T^m y(t)| \leq |\lambda|^m M\mu^m \frac{(b-a)^m}{m!} \ \|x(t) - y(t)\|$$

With the restriction that $\lambda = 1$ and $M(b-a) < 1$, we obtain that T^m is a contraction mapping on X.

We have to show that $(S+T)$ maps $\overline{B}_r$ into itself and is a contraction. Now

$$|(Sx)(t) + (Tx)(t)| \leq |f\big(t, x(t)\big) + \lambda \int_a^b k_1(t,s) g\big(s, x(s)\big) ds$$

$$+ \int_0^t k_2(t,s) h(s, x(s)) ds \,|$$

$$\leq \big|f\big(t, x(t)\big)\big| + \lambda \int_a^b |k_1(t,s)| \, |g(s, x(s))| ds + \int_0^t |k_2(t,s)| |h(s, x(s))| ds$$

$$|(Sx)(t) + (Tx)(t)| \leq \big|f\big(t, x(t)\big)\big| + \lambda k_1^* \big|g\big(t, x(t)\big)\big| + k_2^* \big|h\big(t, x(t)\big)\big| \tag{3.4}$$

We have the following inequalities

$$\big|g\big(t, x(t)\big)\big| = \big|\, g\big(t, x(t)\big) - g(t,0) + g(t,0)\big|$$

$$\leq \big|g\big(t, x(t)\big) - g(t,0)\big| + |g(t,0)|$$

$$\leq \mu |x(t)| + |g(t,0)| \tag{3.5}$$

Similarly

$$\big|h\big(t, x(t)\big)\big| = \big|\, h\big(t, x(t)\big) - h(t,0) + h(t,0)\big|$$

$$\leq \big|h\big(t, x(t)\big) - h(t,0)\big| + |h(t,0)|$$

$$\leq \xi |x(t)| + |h(t,0)| \tag{3.6}$$

From (3.5) and (3.6) we get,

$$|(Sx)(t) + (Tx)(t)| \leq \big|f\big(t, x(t)\big)\big| + \lambda k_1^* \big|g\big(t, x(t)\big)\big| + k_2^* \big|h\big(t, x(t)\big)\big|$$

$$\leq \big|f\big(t, x(t)\big)\big| + \lambda\mu k_1^* |x(t)| + \lambda k_1^* |g(t,0)| + \xi k_2^* |x(t)| + k_2^* |h(t,0)|$$

Since by assumption

$$\big|f\big(t, x(t)\big)\big| + \lambda k_1^* |g(t,0)| + k_1^* |h(t,0)| \leq r(1 - \lambda\mu k_1^* - \xi k_2^*).$$

We have

$$|(Sx)(t)+(Tx)(t)| \le r(1-\lambda\mu k_1^*-\xi k_2^*)+\lambda\mu k_1^* r+\xi k_2^* r=r$$

That is

$$|(Sx)(t)+(Tx)(t)| \le r$$

This shows that

$|(Ux)(t)+(Vx)(t)| \in \overline{B}_r.$

Therefore $S+T$ maps $\overline{B}_r$ into itself.

Also

$$|(Sx)(t)+(Tx)(t)-(Sy)(t)-(Ty)(t)| = |(Sx)(t)-(Sy)(t)+\ (Tx)(t)-(Ty)(t)|$$

$$= |f\big(t,x(t)\big)+\int_0^\infty k_1(t,s)g\big(s,x(s)\big)ds - f\big(t,y(t)\big) - \int_0^\infty k_1(t,s)g\big(s,y(s)\big)ds + \int_0^t k_2(t,s)h\big(s,x(s)\big)ds - \int_0^t k_2(t,s)h\big(s,y(s)\big)ds|$$

$$= |f\big(t,x(t)\big)-f\big(t,y(t)\big) + \int_0^t k_1(t,s)\big[g\big(s,x(s)\big)-g\big(s,y(s)\big)\big]ds - \int_0^\infty k_2(t,s)[h\big(s,x(s)\big)-h\big(s,y(s)\big)]ds|$$

$$\le |f(t,x(t)-f(t,y(t)| + \int_0^t |k_1(t,s)|\big|g\big(s,x(s)\big)-g\big(s,y(s)\big)\big|ds - \int_0^\infty |k_2(t,s)||h\big(s,x(s)\big)-h\big(s,y(s)\big)|ds$$

$$\le |f(t,x(t)-f(t,y(t)| + k_1^*\big|g\big(t,x(t)\big)-g\big(t,y(t)\big)\big| + k_2^*|h\big(t,x(t)\big)-h\big(t,y(t)\big)\big|$$

$$\le r|x(t)-y(t)|+\lambda\mu k_1^*|x(t)-y(t)|+\xi k_2^*|x(t)-y(t)|$$

$$\le (r+\lambda\mu k_1^*+\xi k_2^*)\ |x(t)-y(t)|$$

Since $(r + \mu k_1^* + \xi k_2^*) < 1$ we have $S + T$ is a contraction mapping on $\overline{B}_r$.

Therefore there exists a unique solution of (3.1). This completes the proof of the theorem.

Conclusion

To conclude, the main objective of this study was finding the existence of the solution of integral equations particularly Fredholm-Volterara integral equations. In order to achieve this we described some theorems and related applications have been demonstrated in order to illustrate the reliability. Our application discussed here was the basis of Theorem 1.9. Finally we can say that the generalization of the theorem can be used in further cases and the corollary of the Theorem 2.1 will be more convenient in finding the existence of differential and integral equations.

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